# 6. Jednoduché goniometrické rovnice

ZÁKLADNÍ GONIOMETRICKÁ ROVNICE – rce tvaru

$g(x) = a$ kde $g$ ... jedna z goniometrických funkcí sin, cos, tg, cotg

(rce) $\sin x = 1$ $\quad \operatorname{cotg} x = 1$
$\cos x = -\frac{\sqrt{3}}{2}$ $\quad \operatorname{tg} x = -\sqrt{3}$

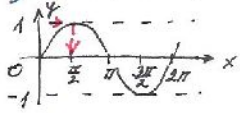
$x$ ... proměnná $(O = \mathbb{R})$
$a$ ... reálné číslo

ZÁKLADNÍ ŘEŠENÍ goniom. rce – množina kořenů v intervalu $\langle 0, 2\pi \rangle$

Příklady

① Řešte v $\mathbb{R}$ TYP: využití grafů

a) $\sin x = 1$



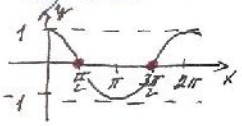

$x = \frac{\pi}{2}$ $(= 90°)$

obecně

$x = \frac{\pi}{2} + 2k\pi \quad k \in \mathbb{Z}$

$\mathcal{K} = \bigcup_{k \in \mathbb{Z}} \{\frac{\pi}{2} + 2k\pi\}$

b) $\cos x = 0$




$x = \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}$

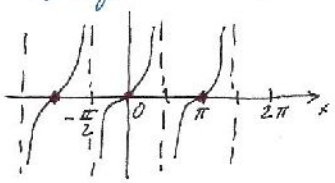
$x = \frac{\pi}{2}(1 + 2k)$

$x = (2k+1)\frac{\pi}{2}$

LICHÉ NÁSOBKY $\frac{\pi}{2}$

$\mathcal{K} = \bigcup_{k \in \mathbb{Z}} \{(2k+1)\frac{\pi}{2}\}$

c) $\operatorname{tg} x = 0$ $\quad [x \neq \frac{\pi}{2} + k\pi]$

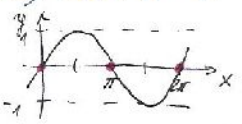



$x = 0 + k\pi \quad k \in \mathbb{Z}$

$x = k\pi$ NÁSOBKY $\pi$

$\mathcal{K} = \bigcup_{k \in \mathbb{Z}} \{k\pi\}$

zú d) $\sin x = 0$

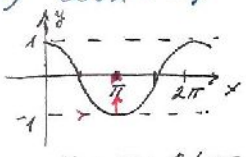



$x = 0 + k\pi \quad k \in \mathbb{Z}$

$x = k\pi$

$\mathcal{K} = \bigcup_{k \in \mathbb{Z}} \{k\pi\}$

zú e) $\cos x = -1$



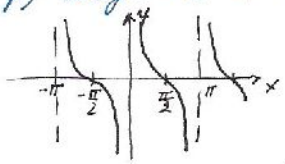

$x = \pi + 2k\pi \quad k \in \mathbb{Z}$

$x = \pi(1 + 2k)$

$x = (2k+1)\pi$

$\mathcal{K} = \bigcup_{k \in \mathbb{Z}} \{(2k+1)\pi\}$

zú f) $\operatorname{cotg} x = 0$ $\quad [x \neq k\pi]$




$x = \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}$

$x = \frac{\pi}{2}(1 + 2k)$

$\mathcal{K} = \bigcup_{k \in \mathbb{Z}} \{(2k+1)\frac{\pi}{2}\}$

② Řešte v $\mathbb{R}$

a) $\sin x = -0{,}5$

(1.řp.) [rce: ⊖ III., IV. kv.
$\sin x' = +0{,}5$, $x' = 30° = \frac{\pi}{6}$]

III. kv. $\quad (x_1 = 180° + x')$

$x_1 = \pi + \frac{\pi}{6} + 2k\pi$

$x_1 = \frac{7\pi}{6} + 2k\pi$

IV. kv. $\quad (x_2 = 360° - x')$

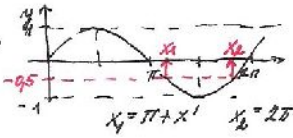
$x_2 = 2\pi - \frac{\pi}{6} + 2k\pi$

$x_2 = \frac{11\pi}{6} + 2k\pi$

$\mathcal{K} = \bigcup_{k \in \mathbb{Z}} \{\frac{7\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi\}$

[ve stupních: $x_1 = 180° + 30° + k \cdot 360°$
$x_1 = 210° + k \cdot 360°$

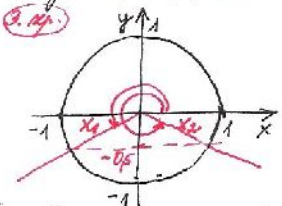
(2.řp.) z grafu




$x_1 = \pi + x'$ $\quad x_2 = 2\pi - x'$ (dále jako 1. řp.)

z jednotkové kružnice

(3.řp.)




$x_1 = \pi + x' + 2k\pi$

$x_2 = 2\pi - x' + 2k\pi$

(dále jako 1. řp.)

$x_2 = 360° - 30° + k \cdot 360°$
$x_2 = 330° + k \cdot 360° \quad k \in \mathbb{Z}$ ]

b) $\cos x = -\frac{\sqrt{2}}{2}$

[kvadr. ⊖ II., III. kvr.
$\cos x' = \frac{\sqrt{2}}{2}$
$x' = 45° = \frac{\pi}{4}$]

II. kvr. $(x_1 = 180° - x')$
$x_1 = \pi - x' + 2k\pi$
$x_1 = \pi - \frac{\pi}{4} + 2k\pi$
$x_1 = \frac{3\pi}{4} + 2k\pi$

III. kvr. $(x_2 = 180° + x')$
$x_2 = \pi + x' + 2k\pi$
$x_2 = \pi + \frac{\pi}{4} + 2k\pi$
$x_2 = \frac{5\pi}{4} + 2k\pi$

$\mathcal{K} = \bigcup_{k\in Z} \{\frac{3\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi\}$

c) $\operatorname{cotg} x = -\frac{\sqrt{3}}{3}$ $[x \neq k\pi]$

[⊖ II., IV. kvr.
$x' = 60° = \frac{\pi}{3}$]

liší, cotg mají periodu $k\pi$ ($k.180°$)!
=> STAČÍ II. kvr.

II. kvr. $x = \pi - x' + k\pi$ $k \in Z$
$x = \pi - \frac{\pi}{3} + k\pi$
$x = \frac{2\pi}{3} + k\pi$

$\mathcal{K} = \bigcup_{k\in Z} \{\frac{2\pi}{3} + k\pi\}$

d) $\operatorname{tg} x = \sqrt{3}$ $[x \neq \frac{\pi}{2} + k\pi]$ (zk)

[⊕ I., (III. kvr.)
$x' = 60° = \frac{\pi}{3}$]

$x = x' + k\pi$ !
$x = \frac{\pi}{3} + k\pi$

$\mathcal{K} = \bigcup_{k\in Z} \{\frac{\pi}{3} + k\pi\}$

e) $\operatorname{cotg} x = -1$ $[x \neq k\pi]$ (zk)

[⊖ II. (IV. kvr. - per. $k\pi$)
$x' = 45° = \frac{\pi}{4}$]

$x = \pi - x' + k\pi$
$x = \pi - \frac{\pi}{4} + k\pi$
$x = \frac{3\pi}{4} + k\pi$

$\mathcal{K} = \bigcup_{k\in Z} \{\frac{3\pi}{4} + k\pi\}$

f) $\sin x = \frac{\sqrt{3}}{2}$ (zk)

[⊕ I., II. kvr.
$x' = 60° = \frac{\pi}{3}$]

I. kvr. $x_1 = x' + 2k\pi$
$x_1 = \frac{\pi}{3} + 2k\pi$

II. kvr. $x_2 = \pi - x' + 2k\pi$
$x_2 = \pi - \frac{\pi}{3} + 2k\pi$
$x_2 = \frac{2\pi}{3} + 2k\pi$

$\mathcal{K} = \bigcup_{k\in Z} \{\frac{\pi}{3} + 2k\pi, \frac{2\pi}{3} + 2k\pi\}$

③ Řeš. v R — TYP SUBSTITUCE

a) $\cos 3x = 0{,}5$

subst. $y = 3x$

$\cos y = 0{,}5$

[⊕ I., IV. kvr.
$y' = 60° = \frac{\pi}{3}$]

I. kvr. $y_1 = y' + 2k\pi$
$y_1 = \frac{\pi}{3} + 2k\pi$

IV. kvr. $y_2 = 2\pi - y' + 2k\pi$
$y_2 = 2\pi - \frac{\pi}{3} + 2k\pi$
$y_2 = \frac{5\pi}{3} + 2k\pi$

ZPĚT K SUBSTITUCI

$3x_1 = \frac{\pi}{3} + 2k\pi$ $\quad 3x_2 = \frac{5\pi}{3} + 2k\pi$
$x_1 = \frac{\pi}{9} + \frac{2}{3}k\pi$ $\quad x_2 = \frac{5\pi}{9} + \frac{2}{3}k\pi$

! PERIODA SE MĚNÍ !

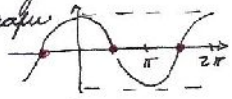
$\mathcal{K} = \bigcup_{k\in Z} \{\frac{\pi}{9} + \frac{2}{3}k\pi, \frac{5\pi}{9} + \frac{2}{3}k\pi\}$

b) $\cos 2x = 0$ (zk)

subst. $y = 2x$

$\cos y = 0$

z grafu




$y = \frac{\pi}{2} + k\pi$ $\quad k \in Z$

vrátíme k subst.

$2x = \frac{\pi}{2} + k\pi$ /:2

$x = \frac{\pi}{4} + k\frac{\pi}{2}$

$\mathcal{K} = \bigcup_{k\in Z} \{\frac{\pi}{4} + k\frac{\pi}{2}\}$

c) $\operatorname{cotg} \pi x = 0$

subst. $\pi x = x$

$\operatorname{cotg} x = 0$

$x = \frac{\pi}{2} + k\pi$

zpět k subst.

$\pi x = \frac{\pi}{2} + k\pi \quad / : \pi$

$x = \frac{1}{2} + k \quad \in \mathbb{D}$

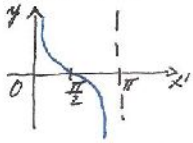
$x = \frac{1+2k}{2}$

$\mathcal{K} = \bigcup_{k \in Z} \left\{ \frac{2k+1}{2} \right\}$

! cotg nuly. pro 0 a násobky $\pi$, tj. $\pi x \neq 0 + k\pi$

$\pi x \neq k\pi$

$x \neq k$




$\mathbb{D} = R - \{k\}$ !

d) $\operatorname{tg} \frac{x}{2} = -1$ 2x!

subst. $\frac{x}{2} = x$

$\operatorname{tg} x = -1$

$[x = 45° = \frac{\pi}{4}]$

⊖ II. kvr. (perioda $k\pi$ => IV. kvr. nemusíme)

$x = \pi - x' + k\pi$

$x = \pi - \frac{\pi}{4} + k\pi$

$x = \frac{3\pi}{4} + k\pi$

zpět k subst.

$\frac{x}{2} = \frac{3\pi}{4} + k\pi$

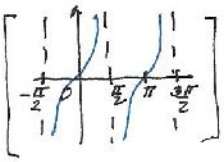
$x = \frac{3\pi}{2} + 2k\pi \in \mathbb{D}$

$\mathcal{K} = \bigcup_{k \in Z} \left\{ \frac{3\pi}{2} + 2k\pi \right\}$

! tg nuly. pro $\frac{\pi}{2}$ a liché nás. $\frac{\pi}{2}$

$\frac{x}{2} \neq \frac{\pi}{2}(2k+1)$

$x \neq \pi(2k+1)$




! $\mathbb{D} = R - \bigcup_{k \in Z} \{(2k+1)\pi\}$

④ Řeším v R

a) $\cos\left(x + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ $\quad O = R$, $\mathbb{D} = R$

subst. $x + \frac{\pi}{4} = x$

$\cos x = -\frac{\sqrt{2}}{2}$

$[x' = 45° = \frac{\pi}{4}]$

⊖ II. kvr., III. kvr. →

$x_1 = \pi - x' + 2k\pi$ $\quad x_2 = \pi + x' + 2k\pi$

$x_1 = \pi - \frac{\pi}{4} + 2k\pi$ $\quad x_2 = \pi + \frac{\pi}{4} + 2k\pi$

$x_1 = \frac{3\pi}{4} + 2k\pi$ $\quad x_2 = \frac{5\pi}{4} + 2k\pi \quad k \in Z$

zpět k subst.

$x_1 + \frac{\pi}{4} = \frac{3\pi}{4} + 2k\pi$ $\quad x_2 + \frac{\pi}{4} = \frac{5\pi}{4} + 2k\pi$

$x_1 = \frac{3\pi}{4} - \frac{\pi}{4} + 2k\pi$ $\quad x_2 = \frac{5\pi}{4} - \frac{\pi}{4} + 2k\pi$

$x_1 = \frac{2\pi}{4} + 2k\pi$ $\quad x_2 = \frac{4\pi}{4} + 2k\pi$

$x_1 = \frac{\pi}{2} + 2k\pi$ $\quad x_2 = \pi + 2k\pi$

$[x_1 = \frac{\pi}{2}(1+4k)]$ nemá vhodné $\quad x_2 = \pi(1+2k)$

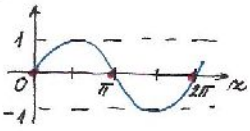
$x_2 = (2k+1)\pi$ liché nás. $\pi$

$\mathcal{K} = \bigcup_{k \in Z} \left\{ \frac{\pi}{2} + 2k\pi, \underbrace{\pi + 2k\pi}_{\text{zkr. } (2k+1)\pi} \right\}$

b) $\sin\left(\frac{\pi}{3} - x\right) = 0$ 2x! $\quad O = R$, $\mathbb{D} = R$

subst. $x = \frac{\pi}{3} - x$

$\sin x = 0$ (z grafu)




$x = 0 + k\pi$

$x = k\pi \quad k \in Z$

zpět k subst.

$\frac{\pi}{3} - x = k\pi$

$-x = -\frac{\pi}{3} + k\pi$

$x = \frac{\pi}{3} \ominus k\pi$

(něk. napíšeme $x = \frac{\pi}{3} - k\pi$)

FÍGL! přeznačíme $k \to k'$ a položíme nové $k = -k'$!

$x = \frac{\pi}{3} \oplus k\pi$

$\mathcal{K} = \bigcup_{k \in Z} \left\{ \frac{\pi}{3} + k\pi \right\}$

[ NEBO

$\frac{\pi}{3} - x = k\pi$

$\frac{\pi}{3} - k\pi = x$

$x = \frac{\pi}{3} - k\pi$

$(x = \frac{\pi}{3} - k'\pi)$

FÍGL! $-k \to k'$ $\Rightarrow -k' = k$

$x = \frac{\pi}{3} + k\pi$ ]

⑤ Řešte v $\mathbb{R}$

a) $2\sin(3x+\pi) = -1$

substituce $x = 3x+\pi$

$2\sin x = -1 \quad /:2$

$\sin x = -\frac{1}{2}$

$[x' = 30° = \frac{\pi}{6}] \ominus$ III., IV. kvadrant

III. kvadrant

$x_1 = \pi + x' + 2k\pi$

$x_1 = \pi + \frac{\pi}{6} + 2k\pi$

$x_1 = \frac{7\pi}{6} + 2k\pi$

IV. kvadrant

$x_2 = 2\pi - x' + 2k\pi$

$x_2 = 2\pi - \frac{\pi}{6} + 2k\pi$

$x_2 = \frac{11\pi}{6} + 2k\pi \quad k \in \mathbb{Z}$

zpět k substituci

$3x_1 + \pi = \frac{7\pi}{6} + 2k\pi$

$3x_1 = \frac{7\pi}{6} - \pi + 2k\pi$

$3x_1 = \frac{\pi}{6} + 2k\pi$

$x_1 = \frac{\pi}{18} + \frac{2}{3}k\pi$

$3x_2 + \pi = \frac{11\pi}{6} + 2k\pi$

$3x_2 = \frac{11\pi}{6} - \pi + 2k\pi$

$3x_2 = \frac{5\pi}{6} + 2k\pi$

$x_2 = \frac{5\pi}{18} + \frac{2}{3}k\pi$

$\mathcal{K} = \bigcup_{k\in\mathbb{Z}} \left\{ \frac{\pi}{18} + \frac{2}{3}k\pi ,\ \frac{5\pi}{18} + \frac{2}{3}k\pi \right\}$

ZKOUŠKA:

$L_1 = 2\sin\left(3\left(\frac{\pi}{18} + \frac{2}{3}k\pi\right) + \pi\right) =$

$= 2\sin\left(\frac{\pi}{6} + \underbrace{2k\pi}_{\text{perioda - vynecháme}} + \pi\right) =$

$= 2\sin\left(\frac{\pi}{6} + \pi\right) = 2\sin\frac{7\pi}{6} =$ [7. 30°]

$= 2\sin 210° = -2\sin 30° =$

$\left[\begin{matrix}\text{III. kvadrant } \ominus \\ x' = 30°\end{matrix}\right] = -2\cdot\frac{1}{2} = -1$

$P_1 = -1 \qquad L_1 = P_1$

$L_2 = 2\sin\left(3\left(\frac{5\pi}{18} + \frac{2}{3}k\pi\right) + \pi\right) =$

$= 2\sin\left(\frac{5\pi}{6} + \underbrace{2k\pi}_{\text{perioda}} + \pi\right) =$

$= 2\sin\left(\frac{5\pi}{6} + \pi\right) = 2\sin\frac{11\pi}{6} =$ [11. 30°]

$= 2\sin 330° = -2\sin 30° =$

$\left[\begin{matrix}\text{IV. kvadrant } \ominus \\ x' = 30°\end{matrix}\right] = -2\cdot\frac{1}{2} = -1$

$P_2 = -1 \qquad L_2 = P_2$

b) $\cos\left(\frac{\pi}{4} - 2x\right) = -\frac{\sqrt{2}}{2}$ $\quad \mathcal{O} = \mathbb{R}$, $\mathcal{D} = \mathbb{R}$

substituce $x = \frac{\pi}{4} - 2x$

$\cos x = -\frac{\sqrt{2}}{2}$

$[x' = 45° = \frac{\pi}{4}] \ominus$ II., III. kvadrant

II. kvadrant

$x_1 = \pi - x' + 2k\pi$

$x_1 = \pi - \frac{\pi}{4} + 2k\pi$

$x_1 = \frac{3\pi}{4} + 2k\pi$

III. kvadrant

$x_2 = \pi + x' + 2k\pi$

$x_2 = \pi + \frac{\pi}{4} + 2k\pi$

$x_2 = \frac{5\pi}{4} + 2k\pi$

zpět k substituci

$\frac{\pi}{4} - 2x = \frac{3\pi}{4} + 2k\pi$

$-2x = \frac{3\pi}{4} - \frac{\pi}{4} + 2k\pi$

$-2x = \frac{\pi}{2} + 2k\pi$

$x = -\frac{\pi}{4} - k\pi$ FIGL - přičtu nač. k

$x = -\frac{\pi}{4} + k\pi$ / + 2π FIGL perioda

$x = -\frac{\pi}{4} + 2\pi + k\pi$ (360°)

$x = \frac{7\pi}{4} + k\pi$

$\frac{\pi}{4} - 2x = \frac{5\pi}{4} + 2k\pi$

$-2x = \frac{5\pi}{4} - \frac{\pi}{4} + 2k\pi$

$-2x = \pi + 2k\pi$

$2x = -\pi - 2k\pi$

$x = -\frac{\pi}{2} - k\pi$ FIGL - přičtu nač. k

$x = -\frac{\pi}{2} + k\pi$ perioda / + 2π FIGL

$x = -\frac{\pi}{2} + 2\pi + k\pi$

$x = \frac{3\pi}{2} + k\pi$

$\mathcal{K} = \bigcup_{k\in\mathbb{Z}} \left\{ \frac{7\pi}{4} + k\pi ,\ \frac{3\pi}{2} + k\pi \right\}$

! POZNÁMKY

- někdy se nechávají nápomocné úhly ve II. kv.

$\mathcal{K} = \bigcup_{k\in\mathbb{Z}} \left\{ -\frac{\pi}{4} + k\pi ,\ -\frac{\pi}{2} + k\pi \right\}$

- pokud chceme hladké $\Rightarrow -\frac{\pi}{4} + 2\pi$ , $-\frac{\pi}{2} + 2\pi$ přičteme periodu $2\pi$

[u $\operatorname{tg}$, $\operatorname{cotg}$ JEN $+\pi$]

c) $\operatorname{tg}\left(-x+\frac{\pi}{6}\right)=\sqrt{3}$

DÚ včetně zkoušky

subst. $u=-x+\frac{\pi}{6}$

$\operatorname{tg} u=\sqrt{3}$

$[u'=60°=\frac{\pi}{3}]$ ⊕ I. kv. (III. kv. nemůžeme, ale perioda $k\pi$)

$u=\frac{\pi}{3}+k\pi \quad k\in Z$

zpět k subst.

$-x+\frac{\pi}{6}=\frac{\pi}{3}+k\pi$

$-x=\frac{\pi}{3}-\frac{\pi}{6}+k\pi$

$-x=\frac{\pi}{6}+k\pi$

$x=-\frac{\pi}{6}-k\pi$ přičm. k,

$x=-\frac{\pi}{6}+k\pi$

$x=-\frac{\pi}{6}+\pi+k\pi$

$x=\frac{5\pi}{6}+k\pi$

$\mathcal{K}=\bigcup_{k\in Z}\left\{-\frac{\pi}{6}+k\pi\right\}$

nebo

$\mathcal{K}=\bigcup_{k\in Z}\left\{\frac{5\pi}{6}+k\pi\right\}$

ZKOUŠKA:

$L=\operatorname{tg}\left(-\frac{5\pi}{6}+k\pi+\frac{\pi}{6}\right)=$

$=\operatorname{tg}\left(-\frac{2\pi}{3}+k\pi\right)=\operatorname{tg}\left(-\frac{2\pi}{3}\right)$

$=-\operatorname{tg}\frac{2\pi}{3}=-\operatorname{tg}120°=-(-\operatorname{tg}60°)=$

$[2\cdot 60°]$ $[$II. kv. ⊖, $x'=60°]$

$=+\operatorname{tg}60°=\sqrt{3}$

$P=\sqrt{3} \qquad L=P$

[obdobně pro $\frac{5\pi}{6}+k\pi$]

TYP: substituce → kv. rce

⑥ Řešte v R

a) $2\cos^2 x-7\cos x+3=0$ $\mathcal{D}=R$, $\mathcal{O}=R$

subst. $y=\cos x$

$2y^2-7y+3=0$

$(a=2,\ b=-7,\ c=3)$ [kv. rce $x_{1/2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$]

$y_{1/2}=\frac{7\pm\sqrt{49-4\cdot2\cdot3}}{4}=\frac{7\pm\sqrt{49-24}}{4}=\frac{7\pm5}{4}$

$y_1=\frac{7+5}{4}=\frac{12}{4}=3 \qquad y_2=\frac{7-5}{4}=\frac{1}{2}$

zpět k substituci

$\cos x_1=3$

NEMÁ ŘEŠENÍ

$[\cos x\in\langle-1,1\rangle]$

$\mathcal{K}_1=\emptyset$

$\cos x=\frac{1}{2}$

$[$I., IV. kv. + $x'=60°=\frac{\pi}{3}]$

I. kv. $x_1=\frac{\pi}{3}+2k\pi$

IV. kv. $x_2=2\pi-x'+2k\pi$

$x_2=2\pi-\frac{\pi}{3}+2k\pi$

$x_2=\frac{5\pi}{3}+2k\pi$

$\mathcal{K}=\bigcup_{k\in Z}\left\{\frac{\pi}{3}+2k\pi,\ \frac{5\pi}{3}+2k\pi\right\}$

NUTNÉ PERFEKTNÍ ZÁPISY!

b) $2\sin^2 x+3\sin x+1=0$ $\mathcal{D}=R$, $\mathcal{O}=R$

DÚ

subst. $y=\sin x$

$2y^2+3y+1=0$

$(a=2,\ b=3,\ c=1)$

$y_{1/2}=\frac{-3\pm\sqrt{9-4\cdot2\cdot1}}{4}=\frac{-3\pm1}{4}=\begin{cases}-\frac{4}{4}=-1\\ -\frac{2}{4}=-\frac{1}{2}\end{cases}$

zpět k subst.

$\sin x=-\frac{1}{2}$

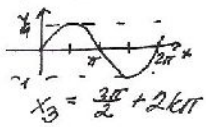
$[x'=30°=\frac{\pi}{6}$, ⊖ III., IV. kv.]

III. kv. $x_1=\pi+x'+2k\pi$

$x_1=\pi+\frac{\pi}{6}+2k\pi$

$x_1=\frac{7\pi}{6}+2k\pi$

$\sin x=-1$



$x_3=\frac{3\pi}{2}+2k\pi$

IV. kv. $x_2=2\pi-x'+2k\pi$

$x_2=2\pi-\frac{\pi}{6}+2k\pi$

$x_2=\frac{11\pi}{6}+2k\pi$

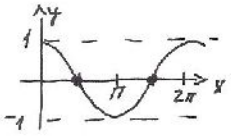
$\mathcal{K}=\bigcup_{k\in Z}\left\{\frac{7\pi}{6}+2k\pi,\ \frac{11\pi}{6}+2k\pi,\ \frac{3\pi}{2}+2k\pi\right\}$

⑦ řešte v $R$ — TYP: součinový tvar

a) $\cos^2 x - \cos x = 0$ — $\mathcal{O}=R$, $\mathcal{D}=R$

$\cos x(\cos x - 1) = 0$

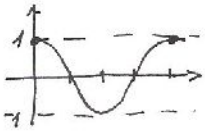
$\cos x = 0$




$x_1 = \frac{\pi}{2} + k\pi$

$x_1 = \frac{\pi}{2}(1+2k)$

$\cos x = 1$




$x_2 = 0 + 2k\pi$

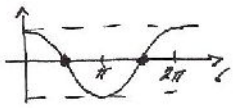
$x_2 = 2k\pi$

$\mathcal{K} = \bigcup_{k\in Z} \{\frac{\pi}{2}(2k+1),\ 2k\pi\}$

b) $\cos^2 x + \cos x = 0$ — $\mathcal{O}=R$, $\mathcal{D}=R$ — DÚ

$\cos x(\cos x + 1) = 0$

$\cos x = 0$




$x_1 = \frac{\pi}{2} + k\pi$

$x_1 = \frac{\pi}{2}(2k+1)$

$\cos x + 1 = 0$

$\cos x = -1$

$x_2 = \pi + 2k\pi$

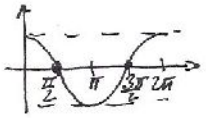
$x_2 = \pi(2k+1)$

$\mathcal{K} = \bigcup_{k\in Z} \{\frac{\pi}{2}(2k+1),\ \pi(2k+1)\}$

c) $2\cos^2 x - \cos x = 0$ — $\mathcal{O}=R$, $\mathcal{D}=R$

$\cos x(2\cos x - 1) = 0$

$\cos x = 0$




$x_1 = \frac{\pi}{2} + k\pi$

$x_1 = \frac{\pi}{2}(1+2k)$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

[I., IV. kvr. ∢, $x' = 60° = \frac{\pi}{3}$]

I. kvr.

$x_2 = x' + 2k\pi$

$x_2 = \frac{\pi}{3} + 2k\pi$

IV. kvr.

$x_3 = 2\pi - x' + 2k\pi$

$x_3 = 2\pi - \frac{\pi}{3} + 2k\pi$

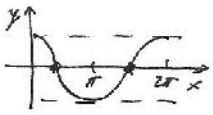
$x_3 = \frac{5\pi}{3} + 2k\pi$

$\mathcal{K} = \bigcup_{k\in Z} \{\frac{\pi}{2}(2k+1),\ \frac{\pi}{3} + 2k\pi,\ \frac{5\pi}{3} + 2k\pi\}$

d) $\cos^2 x - 2\cos x = 0$ — $\mathcal{O}=R$, $\mathcal{D}=R$ — DÚ

$\cos x(\cos x - 2) = 0$

$\cos x = 0$




$x_1 = \frac{\pi}{2} + k\pi$

$x_1 = \frac{\pi}{2}(2k+1)$

$\cos x - 2 = 0$

$\cos x = 2$

NEMÁ ŘEŠ.

[$y = \cos x$, $\mathcal{H} = \langle -1, 1\rangle$]

$\mathcal{K} = \bigcup_{k\in Z} \{\frac{\pi}{2}(2k+1)\}$

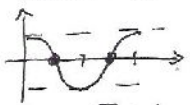
⑧ řešte v $R$

a) $\cos x = \sin 2x \cdot \cos x$ — $\mathcal{O}=R$, $\mathcal{D}=R$

$\cos x - \sin 2x \cdot \cos x = 0$

$\cos x(1 - \sin 2x) = 0$

$\cos x = 0$



$x_1 = \frac{\pi}{2} + k\pi$

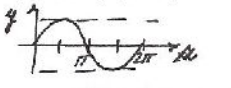
$x_1 = \frac{\pi}{2}(2k+1)$

$1 - \sin 2x = 0$

$\sin 2x = 1$

subst. $z = 2x$

$\sin z = 1$




$z = \frac{\pi}{2} + 2k\pi$

zpět k substituci

$2x = \frac{\pi}{2} + 2k\pi \quad /:2$

$x = \frac{\pi}{4} + k\pi$

$[x = \frac{\pi}{4}(4k+1)]$

$\mathcal{K} = \bigcup_{k\in Z} \{\frac{\pi}{2}(2k+1),\ \frac{\pi}{4} + k\pi\}$

$= \frac{\pi}{4}(4k+1)$

b) $\operatorname{cotg} 2x = \operatorname{tg} 3x \cdot \operatorname{cotg} 2x$

$\operatorname{cotg} 2x - \operatorname{tg} 3x \cdot \operatorname{cotg} 2x = 0$

$\operatorname{cotg} 2x\,(1 - \operatorname{tg} 3x) = 0$

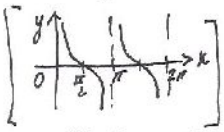
/

$\operatorname{cotg} 2x = 0$

subst. $x = 2x$

$\operatorname{cotg} x = 0$

$x = \frac{\pi}{2} + k\pi \quad k \in Z$




zpět k subst.

$2x = \frac{\pi}{2} + k\pi$

$x_1 = \frac{\pi}{4} + k\frac{\pi}{2}$

$x_1 = \frac{\pi}{4}(1 + 2k)$

\

$1 - \operatorname{tg} 3x = 0$

$\operatorname{tg} 3x = 1$

subst. $x = 3x$

$\operatorname{tg} x = 1$

$[x' = 45°, \text{ I. (III) kv.}]$

1. kv.

$x = x' + k\pi \quad k \in Z$

$x = \frac{\pi}{4} + k\pi$

zpět k subst.

$3x_2 = \frac{\pi}{4} + k\pi$

$x_2 = \frac{\pi}{12} + k\frac{\pi}{3}$

$(x_2 = \frac{\pi}{12}(1 + 4k))$

! D

[ [ $\operatorname{tg}$ nedef. pro $90° + k \cdot 180°$, ] tj. $\frac{\pi}{2} + k\pi \neq x$ ]

nedef. $\operatorname{tg} 3x$ nedef. pro

$3x \neq \frac{\pi}{2} + k\pi \quad k \in Z$

$x \neq \frac{\pi}{6} + k\frac{\pi}{3}$

[ $\operatorname{cotg}$ nedef. pro $0° + k \cdot 180°$, tj. $k\pi$ ]

nedef. $\operatorname{cotg} 2x$ nedef. pro

$2x \neq k\pi \quad k \in Z$

$x \neq k\frac{\pi}{2}$

$D = R - \cup \{\frac{\pi}{6} + k\frac{\pi}{3},\ k\frac{\pi}{2}\}$

NEURČÍME-LI D => ZKOUŠKA !
(ZKUSTE SI)

c) $\sin 2x = \cos 3x \cdot \sin 2x$ $\quad O = R$

DÚ $\sin 2x - \cos 3x \cdot \sin 2x = 0$ $\quad D = R$

$\sin 2x\,(1 - \cos 3x) = 0$

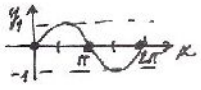
/

$\sin 2x = 0$

subst. $x = 2x$

$\sin x = 0$

$x = k\pi$




zpět k subst.

$2x = k\pi \quad k \in Z$

$x_1 = k\frac{\pi}{2}$

$K = \cup \{k\frac{\pi}{2},\ \frac{2}{3}k\pi\}$ $\quad k \in Z$

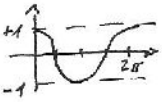
\

$1 - \cos 3x = 0$

$\cos 3x = 1$

subst. $x = 3x$

$\cos x = 1$




$x = 0 + 2k\pi$

$x = 2k\pi$

zpět k subst.

$3x = 2k\pi$

$x_2 = \frac{2}{3}k\pi$

$(x_2 = 2k\frac{\pi}{3})$

ZKUSTE SI ZLOUŠKU !

⑨ Řešte pro $x \in \langle 0°, 360° \rangle$ TYP: VŠECHNA ŘEŠENÍ V INTERVALU

a) $\sin x = -0{,}25$ KALKULAČKY - DEG

[ ⊖ III., IV. kv. $\quad D = \langle 0°, 360° \rangle$

$\sin x' = -0{,}25$

$x' = 14{,}48° = 14°29'$ ]

III. kv. $x_1 = 180° + 14°29'$

$x_1 = 194°29'$

IV. kv. $x_2 = 360° - 14°29'$

$x_2 = 345°31'$

NEPÍŠEME PERIODU - není D

b) $2\cos x = \frac{1}{3}$

DÚ $\cos x = \frac{1}{6}$ $\quad D = \langle 0°, 360° \rangle$

[ ⊕ I., IV. kv. $\quad \cos x' = \frac{1}{6}$

$x' = 80{,}4059° = 80°24'$ ]

I. kv. $x_1 = x' = 80°24'$

IV. kv. $x_2 = 360° - x'$

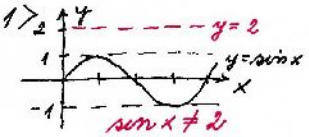
$x_2 = 360° - 80°24'$

$x_2 = 279°36'$

c) $\sin x = 2$ !

$y = \sin x$ } ale $\mathcal{H} = \langle -1,1 \rangle$ !
$y = 2$ } (obor hodnot y)

$\Leftrightarrow 2 \notin \langle -1,1 \rangle_2$




$\mathcal{K} = \emptyset$

d) $\lg x = 3$

⊕ I., III. kv. (musíme obě hodnoty)

[ $x' = 71,5651° \doteq 71°34'$ ]

| I. kv. | III. kv. |
|---|---|
| $x_1 = x'$ | $x_2 = 180° + x'$ |
| $x_1 = 71°34'$ | $x_2 = 251°34'$ |

[ mámu kružil v $\langle 0°, 360° \rangle$, tj. musíme vypsat všechna řešení v tomto intervalu ]

e) $\operatorname{cotg} x = -2$

DÚ ⊖ II., IV. kv. [ NA KALK. $\operatorname{cotg} x = \frac{1}{\operatorname{tg} x}$ ]

[ $\operatorname{cotg} x' = -2$
$x' = 26,5650°$
$x' = 26°34'$ ]

| II. kv. | IV. kv. |
|---|---|
| $x_1 = 180° - x'$ | $x_2 = 360° - x'$ |
| $x_1 = 180° - 26°34'$ | $x_2 = 360° - 26°34'$ |
| $x_1 = 153°26'$ | $x_2 = 333°26'$ |

$\mathcal{K} = \{153°26', 333°26'\}$

f) $\operatorname{tg} x = 0,5$

DÚ ⊕ I., III. kv.

[ $\operatorname{tg} x' = 0,5 \Rightarrow x' = 26,5650'$
$x' = 26°34'$ ]

| I. kv. | III. kv. |
|---|---|
| $x_1 = x'$ | $x_2 = 180° + x'$ |
| $x_1 = 26°34'$ | $x_2 = 206°34'$ |

$\mathcal{K} = \{26°34', 206°34'\}$

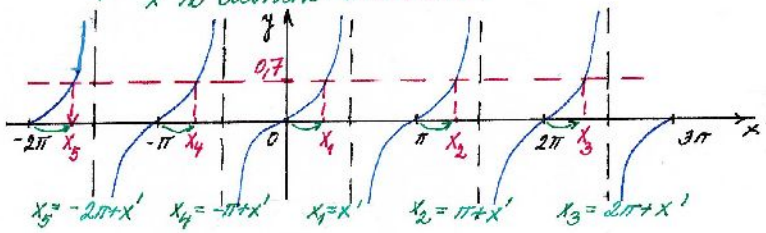
(10) Řešte s neznámou $x \in \langle -2\pi, 3\pi \rangle$ KALKULAČKA RAD

- při tomto zadání musíme určit všechny hodnoty x v daném intervalu

a) $\operatorname{tg} x = 0,7$

[ $\operatorname{tg} x' = 0,7$
kalkul. RAD
$x' = 0,6107$ rad
$x' = 0,611$ rad ]

$x_1 = x' = 0,611$ rad

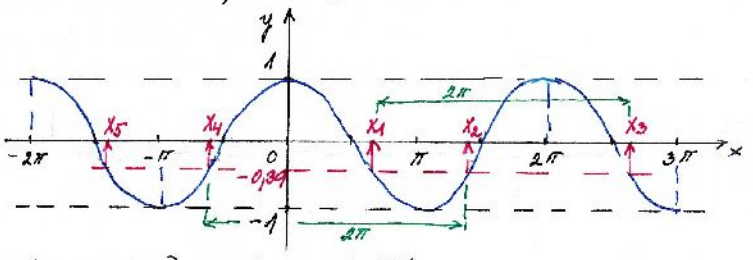



$\mathcal{K} = \{x_1, \pi + x_1, 2\pi + x_1, -\pi + x_1, -2\pi + x_1\}$, kde $x_1 = 0,611$ rad

b) $\cos x = -0,39$

DÚ (1. způs.) $x_1 = 1,971$ NA KALK. (RAD)

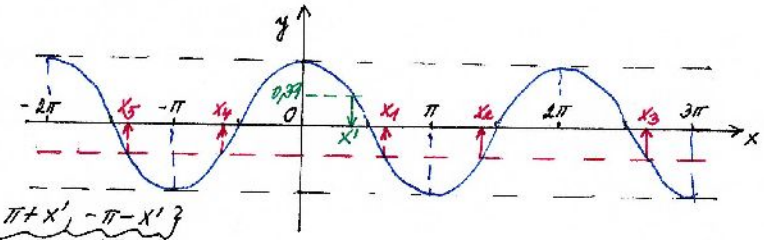
z grafu
$x_4 = -x_1$ $x_5 = x_4 + 2\pi$
$x_2 = x_4 + 2\pi$ $x_3 = x_1 + 2\pi$
$= -x_1 + 2\pi$




$\mathcal{K} = \{x_1, -x_1 + 2\pi, x_1 + 2\pi, -x_1, x_1 - 2\pi\}$ kde $x_1 = 1,971$

(2. způs.)

[ pomocí pomoc. úhlu
$\cos x' = 0,39$
$x' = 1,17$ I. kv. ]




$\mathcal{K} = \{\pi - x', \pi + x', 3\pi - x', -\pi + x', -\pi - x'\}$
kde $x' = 1,17$

[ $3,14 - 1,17 = 1,97 = x_1$ v 1. způs. atd. ] UJASNI SI POSTUPY!